$ASITP$-94-25

A $SU(2)$ Generalized Gauge Field Model With Higgs Mechanism [1]

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Abstract

By means of the non-commutative differential geometry, we construct an $SU(2)$ *generalized gauge field model. It is of* $SU(2) \times \pi_4(SU(2))$ *gauge invariance. We show that this model not only includes the Higgs field automatically on the equal footing with ordinary Yang-Mills gauge potentials but also is stable against quantum correlation.*

PACS numbers: 11.15. -q, 02.40. -k

[1] Work supported in part by The National Natural Science Foundation of China.

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Unlike Yang-Mills gauge fields, Higgs fields and Yukawa couplings seem to be artificial although they play a very important role in modern QFT. Eventually, the price paid for them is the beauty of the gauge principle.

Recently, we have generalized the ordinary Yang-Mills gauge theory in order to take both Lie groups and discrete groups as gauge groups [1,2] and completed an approach to this generalized gauge theory coupled to the fermions in the spirit of non-commutative geometry [3, 4]. We have shown that Higgs fields are such gauge fields with respect to discrete gauge symmetry over 4-dimensional space-time M^4 and the Yukawa couplings between Higgs and fermions may automatically be introduced via generalized covariant derivatives. In this approach, Higgs appears as discrete group gauge fields on the same footing with ordinary Yang-Mills fields over spacetime M^4. In other wards, the beauty of the gauge principle may be regained. Of course, how to understand the physical meaning of the discrete group to be gauged is a most crucial point in this approach. On the other hand, like other approaches [3-10] based upon the non-commutative differential geometry do not survive the standard quantum correlation [11], the approach in [1,2] may also be unstable against the standard quantum correlation unless there is certain special mechanism to guarantee its stability.

In this letter, we will present an $SU(2)$ generalized gauge field model with the Higgs mechanism and show that it will be able to get rid of all those problems. The key point is that we take into account the fourth homotopy group of $SU(2)$ as a discrete gauge group on the footing with the Yang-Mills gauge group $SU(2)$. It is well known that the fourth homotopy group of $SU(2)$ is non-trivial, $\pi_4(SU(2)) = Z_2$, i.e. the gauge transformations of $SU(2)$ may be divided into two different equivalence classes [12]. Once the Yang-Mills fields for the gauge group $SU(2)$ is introduced, the role played by its fourth homotopy group must be taken into account. In view of the generalized Yang-Mills gauge theory [1] based upon the non-commutative differential geometry, we should also introduce the generalized gauge field with respect to this internal discrete group $\pi_4(SU(2))$ due to the fact that the gauge transformations depend on its elements.

Let the elements of $\pi_4(SU(2)) = Z_2 = \{e, r\}$ be $\{U_e, U_r\}$ where U_e represents the equivalence class of the topologically trivial gauge transformations and U_r the topologically non-trivial class modulo topologically trivial gauge transformations. The

model under construction not only includes leptons $\psi(x,h), h \in Z_2$, $SU(2)$ Yang-Mills gauge potentials $A_\mu(x,h)$ and Higgs $\Phi(x,h)$ with respect to this Z_2, but also combines both the Yang-Mills gauge potential and the Higgs on the equal footing as a generalized Yang-Mills gauge potential.

Let us regard those fields as elements of function space on M^4 as well as on $SU(2) \times \pi_4(SU(2))$ and assign them into two sectors according to two elements of $\pi_4(SU(2)) = Z_2 = \{e, r\}$ as follows:

$$
\begin{aligned}
\psi(x,e) = \psi(x) = \begin{pmatrix} L \\ R \end{pmatrix}; \quad \psi(x,r) = \psi^r(x) = \begin{pmatrix} L^r \\ R \end{pmatrix} \\
A_\mu(x,e) = A_\mu(x) = \begin{pmatrix} L_\mu & 0 \\ 0 & 0 \end{pmatrix}; \quad A_\mu(x,r) = A^r_\mu(x) = \begin{pmatrix} L^r_\mu & 0 \\ 0 & 0 \end{pmatrix} \\
\Phi(x,e) = \Phi(x) = \begin{pmatrix} \frac{\mu}{\lambda} & -\phi \\ -\phi^{r\dagger} & \frac{\mu}{\lambda} \end{pmatrix}; \quad \Phi(x,r) = \Phi^r(x);
\end{aligned}
\tag{1}
$$

with $L^r = UL$, $\phi^r = U\phi$, $UU\dagger = 1$, U is a topologically non-trivial $SU(2)$ gauge transformation. In (1), L (R) is the left (right) handed fermion which is an $SU(2)$ doublet (singlet), $L_\mu = -ig\frac{\tau_i}{2}W^i_\mu$ the $SU(2)$-gauge potential, ϕ also an $SU(2)$ doublet, μ and λ two constants.

It should be mentioned, however, that the assignments (1) not only assign the fields to the elements of Z_2 but also imply that all fields are arranged into certain matrices. In fact, this aspect of the arrangements is nothing to do with discrete gauge symmetry but for convenience in the forthcoming calculation. Of course, it must be kept in mind that this is a working hypothesis and sometimes one should avoid certain extra constraints coming from this working hypothesis.

From the general framework in [1], it follows the generalized connection one-form

$$
A(x,h) = A_\mu(x,h)dx^\mu + \frac{\lambda}{\mu}\Phi(x,h)\chi, \quad h \in Z_2, \tag{2}
$$

where χ denotes χ^r, a one form on the function space on $\pi_4(SU(2))$, and the generalized curvature two-form

$$
\begin{aligned}
F(h) &= dA(h) + A(h) \otimes A(h) \\
&= \tfrac{1}{2}F_{\mu\nu}(h)dx^\mu \wedge dx^\nu + \tfrac{\lambda}{\mu}F_{\mu r}(h)dx^\mu \otimes \chi + \tfrac{\lambda^2}{\mu^2}F_{rr}(h)\chi \otimes \chi.
\end{aligned}
\tag{3}
$$

Using the above assignments, we get

$$
\begin{array}{rl}
F(x,e) & = F^r(x,r) \\
& = \frac{1}{2}\begin{pmatrix} L_{\mu\nu} & 0 \\ 0 & 0 \end{pmatrix} dx^\mu \wedge dx^\nu + \frac{\lambda}{\mu}\begin{pmatrix} 0 & -D_\mu\phi \\ -(D_\mu\phi^\dagger)^r & 0 \end{pmatrix} dx^\mu \otimes \chi \\
& + \frac{\lambda^2}{\mu^2}\begin{pmatrix} \phi\phi^\dagger - \frac{\mu^2}{\lambda^2} & 0 \\ 0 & \phi^{r\dagger}\phi^r - \frac{\mu^2}{\lambda^2} \end{pmatrix} \chi \otimes \chi;
\end{array} \tag{4}
$$

where $L_{\mu\nu} = -ig\frac{\tau_i}{2}W^i_{\mu\nu}, \quad D_\mu\phi = \partial_\mu\phi + L_\mu\phi = (\partial_\mu - ig\frac{\tau_i}{2}W^i_\mu)\phi.$

Having these building blocks, we may introduce the generalized gauge invariant Lagrangian with respect to each element of Z_2, then take the Haar integral of them over Z_2 to get the entire Lagrangian of the model. Under certain consideration on the normalization in the Lagrangian, we may get a Lagrangian without any extra constraints among the coupling constants and the mass parameters at the tree level.

For the Lagrangian of the bosonic sector, we have

$$
\begin{array}{rl}
\mathcal{L}_{YM-H}(x,e) & = \mathcal{L}^r_{YM-H}(x,r) \\
& = -\frac{1}{4N_L}Tr(L_{\mu\nu}L^{\mu\nu}) \\
& + \frac{2}{N}\eta\frac{\lambda^2}{\mu^2}Tr(D_\mu\phi(x))(D^\mu\phi(x))^\dagger \\
& - \frac{2}{N}\eta^2\frac{\lambda^4}{\mu^4}Tr(\phi(x)\phi(x)^\dagger - \frac{\mu^2}{\lambda^2})^2 + const;
\end{array} \tag{5}
$$

where N_L and N are normalization constants introduced here to avoid some extra constraints from the matrix arrangement in (1), η a metric parameter defined by $\eta = <\chi, \chi>, \ Dim(\eta) = \mu^2$. The normalization of the coefficients of each term results

$$
N_L = \frac{g^2}{2}, \ N = 2\frac{\lambda^2}{\mu^2}\eta. \tag{6}
$$

For the fermionic sector, the Lagrangian may also be given as follows:

$$
\begin{array}{rl}
\mathcal{L}_F(x,e) & = \mathcal{L}^r_F(x,r) \\
& = i\overline{L}\gamma^\mu(\partial_\mu + L_\mu)L + i\overline{R}\gamma^\mu\partial_\mu R + \lambda(\overline{L}\phi R + \overline{R}\phi^\dagger L).
\end{array} \tag{7}
$$

Then the entire Lagrangian for the model reads:

$$
\mathcal{L}(x) = \frac{1}{2}\sum_{h=e,r}\{\mathcal{L}_F(x,h) + \mathcal{L}_{YM-H}(x,h)\}. \tag{8}
$$

It is remarkable that this is a Lagrangian with the Higgs mechanism of spontaneously symmetry breaking type and the Yukawa couplings included automatically.

In fact, the Higgs potential takes its minimum value at $Tr(\phi\phi^\dagger) = (\frac{\mu}{\lambda})^2$ and the continuous gauge symmetry $SU(2)$ will spontaneously be broken down when the vacuum expectation value is taken as, say,

$$<\phi>= \begin{pmatrix} 0 \\ \frac{\rho_0}{\sqrt{2}} \end{pmatrix}, \tag{9}$$

where $\rho_0 = \sqrt{2}\frac{\mu}{\lambda}$. Now we take the vacuum expectation value of ϕ and introduce a new field $\rho(x)$

$$\phi = \begin{pmatrix} 0 \\ \frac{\rho_0+\rho(x)}{\sqrt{2}} \end{pmatrix}. \tag{10}$$

Then we have in the Lagrangian of the bosonic part

$$\begin{aligned} &Tr\{D_\mu\phi(D_\mu\phi)^\dagger - \eta\tfrac{\lambda^2}{\mu^2}(\phi\phi^\dagger - \tfrac{\mu^2}{\lambda^2})^2\} \\ &= \tfrac{1}{2}\partial_\mu\rho\partial^\mu\rho + \tfrac{g^2}{4}(\rho_0+\rho)^2 W_\mu^- W_\mu^+ + \tfrac{1}{8}g^2(\rho_0+\rho)^2 W_\mu^3 W_\mu^3 \\ &\quad - \eta\tfrac{\lambda^2}{\mu^2}\rho^2(\rho_0^2 + \rho_0\rho + \tfrac{\rho^2}{4}) + const. \end{aligned} \tag{11}$$

It is easy to see that only all gauge bosons $W^\pm$ and W^3 and Higgs ρ become massive:

$$M_W = \frac{1}{2}g\rho_0, \quad M_{Higgs} = 2\sqrt{\eta}. \tag{12}$$

While for the fermions, one of the components of L, the down fermion in the $SU(2)$-doublet, becomes massive with mass μ and others remain massless. If the metric parameter η is free of choice, there do not exist any constraints among the coupling constants. This is different from other approaches [5-10].

Let us now summarize what we have done. Based on a first principle, the generalized gauge principle, we have constructed an $SU(2)$ generalized gauge field model with $\pi_4(SU(2)) = Z_2$ taken as discrete gauge symmetry. The Higgs mechanism is automatically included in this generalized gauge theory model.

It is worthy to point out that there are several advantages in this approach. First, this $\pi_4(SU(2))$ is a most natural and meaningful internal symmetry to be gauged in the model. What we have done here is just to combine the ordinary Yang-Mills gauge theory with the non-commutative differential calculus in the function space on this discrete group to formulate a generalized gauge theory with Higgs and spontaneously symmetry breaking. In other wards, the Higgs mechanism should be introduced automatically on the equal footing with ordinary Yang-Mills gauge fields, if the role played by the

fourth homotopy group of the gauge groups would be taken into account together with the gauge groups themselves at very beginning.

It is even more remarkable that the approach presented here is stable against quantum correlation. One of the reasons is that there are no extra constraints among the parameters at the tree level so that we do not need to pay attention to them in the course of quantization. Another reason may be more essential. Namely, since the Higgs potential is automatically introduced, the $SU(2)$ gauge symmetry should be spontaneously broken down at the tree level in this model. Therefore, this $\pi_4(SU(2)) = Z_2$ symmetry is also broken down as long as the VEV for the Higgs is taken. Consequently, what we got is the same version as an ordinary $SU(2)$ Yang-Mills model with Higgs mechanism and of course we do not need to concern about this $\pi_4(SU(2)) = Z_2$-gauge symmetry when we consider the quantum correlation in the model. Needless to say, this very important point is completely different from other approaches to the Higgs by means of the non-commutative differential geometry. In fact, Connes like approaches [5-10] do not survive the standard quantum correlation [11].

It is clear that the model presented here is not phenomenologically realistic but it can be generalized to other gauge theory models, such as the Weinberg-Salam model and the standard model for the electroweak-strong interaction. In those cases, this approach may also shed the light on the Higgs pattern. Since $\pi_4(SU(N)) = 0,\ N \neq 2$, Higgs mechanism of this type should not appear in the gauge field sectors of $SU(N),\ N \neq 2$. On the other hand, the model may also be generalized to the case of $SU(2)_L \times SU(2)_R$ gauge invariance with $\pi_4(SU(2)_L \times SU(2)_R) = Z_2 \times Z_2'$ and it may be applied to the left-right symmetric model. Furthermore, since the fourth homotopy group of $SU(5)/(SU(3) \times SU(2) \times U(1))$ is also non-trivial, it may play certain role in the SU(5)-GUT together with $\pi_4(SU(3) \times SU(2) \times U(1)) = Z_2$. And all models of this kind may have the same advantages as the one presented in this letter. Especially, all of them should also be stable against quantum correlation.

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